TRIGONOMETRÍA I - EXAMEN PARCIAL

1) $\operatorname{sen}\alpha = -\frac{2}{3}$ y $180^\circ < \alpha < 270^\circ \rightarrow 3^{er}$ cuadrante

$$\operatorname{sen}^2\alpha + \cos^2\alpha = 1$$

$$\left(-\frac{2}{3}\right)^2 + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = 1 - \frac{4}{9} \Rightarrow \cos^2\alpha = \frac{5}{9} \Rightarrow \cos\alpha = \pm\frac{\sqrt{5}}{3} \xrightarrow{3^{er} \text{cuad}}$$

$$\rightarrow \cos\alpha = -\frac{\sqrt{5}}{3}$$

$$\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{-2/3}{-\sqrt{5}/3} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

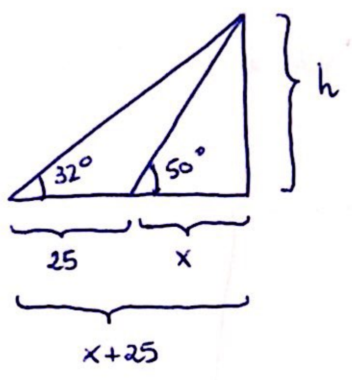
a)

$$\boxed{\begin{array}{lcl} \operatorname{sen}\alpha = -\frac{2}{3} & \longrightarrow & \operatorname{cosec}\alpha = -\frac{3}{2} \\ \cos\alpha = -\frac{\sqrt{5}}{3} & \longrightarrow & \sec\alpha = -\frac{3}{\sqrt{5}} = -\frac{3\sqrt{5}}{5} \\ \operatorname{tg}\alpha = \frac{2\sqrt{5}}{5} & \longrightarrow & \operatorname{cotg}\alpha = \frac{\sqrt{5}}{2} \end{array}}$$

b)

$$\boxed{\begin{array}{l} \operatorname{sen}(90^\circ - \alpha) = \cos\alpha = -\frac{\sqrt{5}}{3} \\ \cos(90^\circ - \alpha) = \operatorname{sen}\alpha = -\frac{2}{3} \\ \operatorname{tg}(90^\circ - \alpha) = \operatorname{cotg}\alpha = \frac{\sqrt{5}}{2} \end{array}}$$

2)



$$\operatorname{tg} 50^\circ = \frac{h}{x} \longrightarrow h = x \cdot \operatorname{tg} 50^\circ$$

$$\operatorname{tg} 32^\circ = \frac{h}{x+25} \longrightarrow h = (x+25) \cdot \operatorname{tg} 32^\circ$$

Igualo: $x \operatorname{tg} 50^\circ = (x+25) \operatorname{tg} 32^\circ$

$$x \operatorname{tg} 50^\circ = x \operatorname{tg} 32^\circ + 25 \cdot \operatorname{tg} 32^\circ$$

$$x(\operatorname{tg} 50^\circ - \operatorname{tg} 32^\circ) = 25 \cdot \operatorname{tg} 32^\circ$$

$$x = \frac{25 \cdot \operatorname{tg} 32^\circ}{\operatorname{tg} 50^\circ - \operatorname{tg} 32^\circ} = 27{,}56 \text{ m}$$

Calculamos la altura:

$$h = x \cdot \operatorname{tg} 50^\circ = 27{,}56 \cdot \operatorname{tg} 50^\circ = \boxed{32{,}84 \text{ m}}$$

③ a) $\operatorname{sen}(180° - \alpha) \cdot \cos(90° - \alpha) - \cos(360° - \alpha) \cdot \cos(180° + \alpha) =$

$= \operatorname{sen}\alpha \cdot \operatorname{sen}\alpha - \cos\alpha \cdot (-\cos\alpha) = \operatorname{sen}^2\alpha + \cos^2\alpha = \boxed{1}$

b) $\cos\frac{14\pi}{3} + \operatorname{sen}\frac{7\pi}{6} = \cos 840° + \operatorname{sen} 210° = \cos 120° + \operatorname{sen} 210° =$

$840° \,|\underline{360°}$
$120 \quad 2$

$= \cos(180° - 60°) + \operatorname{sen}(180° + 30°) =$

$= -\cos 60° - \operatorname{sen} 30° = -\frac{1}{2} - \frac{1}{2} = \boxed{-1}$

④ Teorema de los senos:

$$\frac{30}{\operatorname{sen}25°} = \frac{60}{\operatorname{sen}A} \Rightarrow \operatorname{sen}A = \frac{60 \cdot \operatorname{sen}25°}{30} \Rightarrow A = \arcsin\left(\frac{60 \cdot \operatorname{sen}25°}{30}\right) =$$

$$= \begin{cases} A_1 = 57{,}7° \\ A_2 = 180° - 57{,}7° = \\ \quad = 122{,}3° \end{cases}$$

• Si $A_1 = 57{,}7° \Rightarrow C_1 = 180° - 57{,}7° - 25° = 97{,}3°$

Para calcular c_1 usamos el teorema del coseno:

$$c_1 = \sqrt{a^2 + b^2 - 2ab\cos C_1} = \sqrt{60^2 + 30^2 - 2 \cdot 60 \cdot 30 \cdot \cos 97{,}3°} =$$

$= 70{,}4$ m

• Si $A_2 = 122{,}3° \Rightarrow C_1 = 180° - 122{,}3° - 25° = 32{,}7°$

$$c_2 = \sqrt{60^2 + 30^2 - 2 \cdot 60 \cdot 30 \cdot \cos 32{,}7°} = 38{,}35 \text{ m}$$

Las dos soluciones son:

①	②
a = 60 m	a = 60 m
b = 30 m	b = 30 m
c = 70,4 m	c = 38,35 m
A = 57,7°	A = 122,3°
B = 25°	B = 25°
C = 97,3°	C = 32,7°

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$$\frac{\operatorname{sen}\alpha - \operatorname{sen}^3\alpha}{\cos\alpha - \cos^3\alpha} = \operatorname{cotg}\alpha$$

$$\frac{\operatorname{sen}\alpha - \operatorname{sen}^3\alpha}{\cos\alpha - \cos^3\alpha} = \frac{\operatorname{sen}\alpha\,(1-\operatorname{sen}^2\alpha)}{\cos\alpha\,(1-\cos^2\alpha)} = \frac{\cancel{\operatorname{sen}\alpha}\cdot\cos^2\alpha}{\cancel{\cos\alpha}\cdot\operatorname{sen}^2\alpha} = \frac{\cos\alpha}{\operatorname{sen}\alpha} = \operatorname{cotg}\alpha$$

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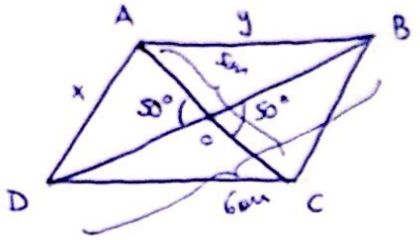
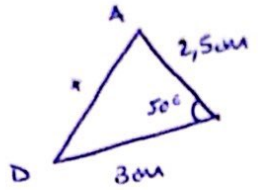


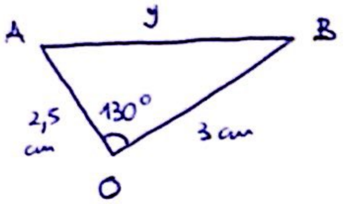
Tomamos el triángulo $\widehat{ADO}$:



Aplico teorema del coseno.

$$x = \sqrt{3^2 + 2,5^2 - 2\cdot 3\cdot 2,5\cdot\cos 50^\circ} = 2,37\text{ cm}$$

Tomamos el triángulo $\widehat{AOB}$:



Volvemos a aplicar teorema del coseno:

$$y = \sqrt{2,5^2 + 3^2 - 2\cdot 2,5\cdot 3\cdot\cos 130^\circ} = 4,99\text{ cm}$$

$$\text{Perímetro} = 2x + 2y = 2\cdot 2,37 + 2\cdot 4,99 = \boxed{14,72\text{ cm}}$$