

FICHA REPASO TRIGONOMETRÍA

- ①
- a) $\text{sen } 175^\circ = \text{sen } (180^\circ - 5^\circ) = \text{sen } 5^\circ = 0,0871$
 - b) $\text{sen } 185^\circ = \text{sen } (180^\circ + 5^\circ) = -\text{sen } 5^\circ = -0,0871$
 - c) $\text{cos } 85^\circ = \text{sen } (90^\circ - 85^\circ) = \text{sen } 5^\circ = 0,0871$
 - d) $\text{sen } 725^\circ = \text{sen } 5^\circ = 0,0871$ $\begin{array}{r} 725 \quad | 360 \\ \underline{5} \quad \quad 2 \end{array}$
 - e) $\text{sen } (-5^\circ) = -\text{sen } 5^\circ = -0,0871$
 - f) $\text{sen } (355^\circ) = \text{sen } (360^\circ - 5^\circ) = -\text{sen } 5^\circ = -0,0871$

- ②
- a) $\text{sen } 180^\circ = \text{sen } (180^\circ - 0^\circ) = \text{sen } 0^\circ = 0$
 - b) $\text{cos } (-120^\circ) = \text{cos } 120^\circ = \text{cos } (180^\circ - 60^\circ) = -\text{cos } 60^\circ = -\frac{1}{2}$
 - c) $\text{tg } (-30^\circ) = -\text{tg } 30^\circ = -\frac{\sqrt{3}}{3}$
 - d) $\text{cotg } 4500^\circ = \text{cotg } 180^\circ = \frac{1}{\text{tg } 180^\circ} = \frac{1}{\text{tg } (180^\circ - 0^\circ)} = \frac{1}{\text{tg } 0^\circ} = \frac{1}{0} = \text{no existe}$ $\begin{array}{r} 4500 \quad | 360 \\ \underline{0900} \quad 12 \end{array}$
 - e) $\text{sec } (-270^\circ) = \text{sec } 90^\circ = \frac{1}{\text{cos } 90^\circ} = \frac{1}{0} = \text{no existe}$
 - f) $\text{cosec } 2700^\circ = \text{cosec } 180^\circ = \frac{1}{\text{sen } 180^\circ} = \frac{1}{\text{sen } 0^\circ} = \frac{1}{0} = \text{no existe}$

③ $65^\circ \rightarrow$

$$\left. \begin{array}{l} \text{sen}^2 \alpha + \text{cos}^2 \alpha = 1 \\ (0,9)^2 + \text{cos}^2 \alpha = 1 \\ \text{cos}^2 \alpha = 1 - 0,81 \\ \text{cos } \alpha = \sqrt{0,19} \\ \text{cos } \alpha = 0,436 \end{array} \right\} \quad \begin{array}{l} \text{cos } 65^\circ = 0,436 \\ \text{tg } 65^\circ = \frac{\text{sen } 65^\circ}{\text{cos } 65^\circ} = 2,077 \end{array}$$

$115^\circ = 180^\circ - 65^\circ \rightarrow$

$$\begin{array}{l} \text{sen } 115^\circ = \text{sen } (180^\circ - 65^\circ) = \text{sen } 65^\circ = 0,906 \\ \text{cos } 115^\circ = \text{cos } (180^\circ - 65^\circ) = -\text{cos } 65^\circ = -0,436 \\ \text{tg } 115^\circ = \text{tg } (180^\circ - 65^\circ) = -\text{tg } 65^\circ = -2,077 \end{array}$$

(son complementarios)

$25^\circ = 90^\circ - 65^\circ \rightarrow$

$$\begin{array}{l} \text{sen } 25^\circ = \text{sen } (90^\circ - 65^\circ) = \text{cos } 65^\circ = 0,436 \\ \text{cos } 25^\circ = \text{sen } 65^\circ = 0,906 \\ \text{tg } 25^\circ = \text{cotg } 65^\circ = \frac{1}{\text{tg } 65^\circ} = \frac{1}{2,077} = 0,481 \end{array}$$

$155^\circ = 180^\circ - 25^\circ \rightarrow$

$$\begin{array}{l} \text{sen } 155^\circ = \text{sen } 25^\circ = 0,436 \\ \text{cos } 155^\circ = -\text{cos } 25^\circ = -0,906 \\ \text{tg } 155^\circ = -\text{tg } 25^\circ = -0,481 \end{array}$$

$$205^\circ = 180^\circ + 25^\circ \longrightarrow \begin{aligned} \text{sen } 205^\circ &= \text{sen}(180^\circ + 25^\circ) = -\text{sen } 25^\circ = -0,436 \\ \cos 205^\circ &= \cos(180^\circ + 25^\circ) = -\cos 25^\circ = -0,906 \\ \text{tg } 205^\circ &= \text{tg}(180^\circ + 25^\circ) = \text{tg } 25^\circ = 2,077 \end{aligned}$$

$$245^\circ = 180^\circ + 65^\circ \longrightarrow \begin{aligned} \text{sen } 245^\circ &= -\text{sen } 65^\circ = -0,906 \\ \cos 245^\circ &= -\cos 65^\circ = -0,436 \\ \text{tg } 245^\circ &= \text{tg } 65^\circ = 2,077 \end{aligned}$$

$$295^\circ = 180^\circ + 115^\circ \longrightarrow \begin{aligned} \text{sen } 295^\circ &= -\text{sen } 115^\circ = -0,906 \\ \cos 295^\circ &= -\cos 115^\circ = -0,436 \\ \text{tg } 295^\circ &= \text{tg } 115^\circ = -2,077 \end{aligned}$$

$$335^\circ = 360^\circ - 25^\circ \longrightarrow \begin{aligned} \text{sen } 335^\circ &= -\text{sen } 25^\circ = -0,436 \\ \cos 335^\circ &= \cos 25^\circ = 0,906 \\ \text{tg } 335^\circ &= -\text{tg } 25^\circ = -0,481 \end{aligned}$$

$$\textcircled{4} \quad \text{sen}^2 \alpha + \cos^2 \alpha = 1$$

$$\left(\frac{3}{5}\right)^2 + \cos^2 \alpha = 1$$

$$\frac{9}{25} + \cos^2 \alpha = 1$$

$$\cos^2 \alpha = \frac{16}{25}$$

$$\cos \alpha = \pm \frac{4}{5} \longrightarrow \cos \alpha = -\frac{4}{5}$$

$$\text{tg } \alpha = \frac{3/5}{-4/5} = -\frac{3}{4}$$

$\text{sen } \alpha = \frac{3}{5}$
$\cos \alpha = -\frac{4}{5}$
$\text{tg } \alpha = -\frac{3}{4}$

- $\text{sen}(90^\circ - \alpha) = \cos \alpha = -\frac{4}{5}$
 $\cos(90^\circ - \alpha) = \text{sen } \alpha = \frac{3}{5}$
 $\text{tg}(90^\circ - \alpha) = \cot \alpha = -\frac{4}{3}$

- $\text{sen}(180^\circ - \alpha) = \text{sen } \alpha = \frac{3}{5}$
 $\cos(180^\circ - \alpha) = -\cos \alpha = \frac{4}{5}$
 $\text{tg}(180^\circ - \alpha) = -\text{tg } \alpha = \frac{3}{4}$

- $\text{sen}(180^\circ + \alpha) = -\text{sen } \alpha = -\frac{3}{5}$
 $\cos(180^\circ + \alpha) = -\cos \alpha = \frac{4}{5}$
 $\text{tg}(180^\circ + \alpha) = \text{tg } \alpha = -\frac{3}{4}$

- $\text{sen}(-\alpha) = -\text{sen } \alpha = -\frac{3}{5}$
 $\cos(-\alpha) = \cos \alpha = -\frac{4}{5}$
 $\text{tg}(-\alpha) = -\text{tg } \alpha = \frac{3}{4}$

$$5) \quad a) \quad \sin 120^\circ - \cos 150^\circ + \frac{\cos 300^\circ - \sin 330^\circ}{\tan 225^\circ}$$

$$\cdot \sin 120^\circ = \sin (180^\circ - 60^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cdot \cos 150^\circ = \cos (180^\circ - 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

$$\cdot \cos 300^\circ = \cos (360^\circ - 60^\circ) = \cos 60^\circ = \frac{1}{2}$$

$$\cdot \sin 330^\circ = \sin (360^\circ - 30^\circ) = -\sin 30^\circ = -\frac{1}{2}$$

$$\cdot \tan 225^\circ = \tan (180^\circ + 45^\circ) = \tan 45^\circ = 1$$

$$\frac{\frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right)}{1} + \frac{\frac{1}{2} - \left(-\frac{1}{2}\right)}{1} = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + 1 = \frac{2\sqrt{3}}{2} + 1 = \sqrt{3} + 1$$

$$b) \quad \frac{\sin 270^\circ \cdot \cos 135^\circ}{(1 + \cos 225^\circ)(1 - \cos 225^\circ)} = \frac{\sin 270^\circ \cdot \cos 135^\circ}{1 - \cos^2 225^\circ} =$$

$$\cdot \sin 270^\circ = \sin (180^\circ + 90^\circ) = -\sin 90^\circ = -1$$

$$\cdot \cos 135^\circ = \cos (180^\circ - 45^\circ) = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$$

$$\cdot \cos 225^\circ = \cos (180^\circ + 45^\circ) = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$$

$$= \frac{-1 \cdot \left(-\frac{\sqrt{2}}{2}\right)}{1 - \left(-\frac{\sqrt{2}}{2}\right)} = \frac{\frac{\sqrt{2}}{2}}{1 + \frac{\sqrt{2}}{2}} = \frac{\frac{\sqrt{2}}{2}}{\frac{2 + \sqrt{2}}{2}} = \frac{\sqrt{2}}{2 + \sqrt{2}}$$

$$c) \quad \cos^2 315^\circ \cdot \left(\frac{\tan 225^\circ}{\sin 90^\circ} - 2 \sin^2 135^\circ \right) = \left(\frac{\sqrt{2}}{2} \right)^2 \cdot \left(1 - 2 \left(\frac{\sqrt{2}}{2} \right)^2 \right) =$$

$$\tan 225^\circ = 1$$

$$\sin 90^\circ = 1$$

$$\cos 315^\circ = \cos (360^\circ - 45^\circ) = \cos 45^\circ = \frac{\sqrt{2}}{2}$$

$$\sin 135^\circ = \sin (180^\circ - 45^\circ) = \frac{\sqrt{2}}{2}$$

$$= \frac{1}{2} \cdot \left(1 - 2 \cdot \frac{1}{2} \right) = \frac{1}{2} \cdot (1 - 1) = 0$$

$$d) \operatorname{sen} \frac{10\pi}{3} - 2 \cos \frac{2\pi}{3} - \cos \frac{3\pi}{6} - 2 \operatorname{tg} \frac{9\pi}{4} =$$

$$= \operatorname{sen} 600^\circ - 2 \cos 120^\circ - \cos 210^\circ - 2 \operatorname{tg} 405^\circ =$$

$$\cdot \operatorname{sen} 600^\circ = \operatorname{sen} 240^\circ = \operatorname{sen} (120^\circ + 60^\circ) = -\operatorname{sen} 60^\circ = -\frac{\sqrt{3}}{2}$$

$$\cdot \cos 120^\circ = \cos (120^\circ - 60^\circ) = -\cos 60^\circ = -\frac{1}{2}$$

$$\cdot \cos 210^\circ = \cos (120^\circ + 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

$$\cdot \operatorname{tg} 405^\circ = \operatorname{tg} 45^\circ = 1$$

$$= -\frac{\sqrt{3}}{2} - 2 \cdot \left(-\frac{1}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right) - 2 = \cancel{-\frac{\sqrt{3}}{2}} + 1 + \cancel{\frac{\sqrt{3}}{2}} - 2 = -1$$

$$e) \operatorname{sen} \frac{17\pi}{6} + \frac{1}{2} + \operatorname{tg} \frac{3\pi}{4} + 2 \cos \frac{11\pi}{6} - 2 \operatorname{sen} \frac{4\pi}{3} =$$

$$= \operatorname{sen} 510^\circ + \frac{1}{2} + \operatorname{tg} 135^\circ + 2 \cos 330^\circ - 2 \operatorname{sen} 240^\circ =$$

$$\cdot \operatorname{sen} 510^\circ = \operatorname{sen} 150^\circ = \operatorname{sen} (180^\circ - 30^\circ) = \operatorname{sen} 30^\circ = \frac{1}{2}$$

$$\cdot \operatorname{tg} 135^\circ = \operatorname{tg} (180^\circ - 45^\circ) = -\operatorname{tg} 45^\circ = -1$$

$$\cdot \cos 330^\circ = \cos (360^\circ - 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\cdot \operatorname{sen} 240^\circ = \operatorname{sen} (180^\circ + 60^\circ) = -\operatorname{sen} 60^\circ = -\frac{\sqrt{3}}{2}$$

$$= \frac{1}{2} + \frac{1}{2} \cdot (-1) + 2 \cdot \frac{\sqrt{3}}{2} - 2 \left(-\frac{\sqrt{3}}{2}\right) = \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} + \frac{2\sqrt{3}}{2} + \frac{2\sqrt{3}}{2} = 2\sqrt{3}$$

$$(6) \quad a) \cos(90^\circ - \alpha) \cdot \cos(180^\circ + \alpha) + \operatorname{sen}(90^\circ - \alpha) \cdot \operatorname{sen}(180^\circ - \alpha) =$$

$$= \operatorname{sen} \alpha \cdot (-\cos \alpha) + \cos \alpha \cdot \operatorname{sen} \alpha = -\operatorname{sen} \alpha \cos \alpha + \cos \alpha \operatorname{sen} \alpha = 0$$

$$b) \operatorname{sen}(180^\circ - \alpha) \cdot \cos(90^\circ - \alpha) + \cos(180^\circ - \alpha) \cdot \cos(180^\circ + \alpha) =$$

$$= \operatorname{sen} \alpha \cdot \operatorname{sen} \alpha + (-\cos \alpha) \cdot (-\cos \alpha) = \operatorname{sen}^2 \alpha + \cos^2 \alpha = 1$$

$$c) \cos(360^\circ - \alpha) + \operatorname{sen}(180^\circ - \alpha) - \operatorname{sen}(360^\circ - \alpha) - \cos(180^\circ + \alpha) =$$

$$= \cos \alpha + \operatorname{sen} \alpha - (-\operatorname{sen} \alpha) - (-\cos \alpha) = \cos \alpha + \operatorname{sen} \alpha + \operatorname{sen} \alpha + \cos \alpha =$$

$$= 2 \operatorname{sen} \alpha + 2 \cos \alpha$$

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a) $\sec^2 \alpha + \operatorname{cosec}^2 \alpha = \sec^2 \alpha \cdot \operatorname{cosec}^2 \alpha$

$$\sec^2 \alpha + \operatorname{cosec}^2 \alpha = \frac{1}{\cos^2 \alpha} + \frac{1}{\sin^2 \alpha} = \frac{\sin^2 \alpha + \cos^2 \alpha}{\cos^2 \alpha \cdot \sin^2 \alpha} = \frac{1}{\cos^2 \alpha \cdot \sin^2 \alpha}$$

$$= \frac{1}{\cos^2 \alpha} \cdot \frac{1}{\sin^2 \alpha} = \sec^2 \alpha \cdot \operatorname{cosec}^2 \alpha$$

b) $\cos^2 \alpha = \frac{\cot g^2 \alpha}{1 + \cot g^2 \alpha}$

$$\frac{\cot g^2 \alpha}{1 + \cot g^2 \alpha} = \frac{\frac{\cos^2 \alpha}{\sin^2 \alpha}}{1 + \frac{\cos^2 \alpha}{\sin^2 \alpha}} = \frac{\frac{\cos^2 \alpha}{\sin^2 \alpha}}{\frac{\sin^2 \alpha + \cos^2 \alpha}{\sin^2 \alpha}} = \frac{\cos^2 \alpha}{1} = \cos^2 \alpha$$

c) $\frac{1 + \tan^2 \alpha}{\cot g \alpha} = \frac{\tan \alpha}{\cos^2 \alpha}$

$$\frac{1 + \tan^2 \alpha}{\cot g \alpha} = \frac{\sec^2 \alpha}{\cot g \alpha} = \frac{\frac{1}{\cos^2 \alpha}}{\frac{\cos \alpha}{\sin \alpha}} = \frac{1}{\cos^2 \alpha} \cdot \frac{\sin \alpha}{\cos \alpha} = \frac{\sin \alpha}{\cos^2 \alpha \cdot \cos \alpha}$$

$$= \frac{1}{\cos^2 \alpha} \cdot \frac{\sin \alpha}{\cos \alpha} = \frac{1}{\cos^2 \alpha} \cdot \tan \alpha = \frac{\tan \alpha}{\cos^2 \alpha}$$

d) $\frac{\sec \alpha - \cos \alpha}{\operatorname{cosec} \alpha - \sin \alpha} = \tan^3 \alpha$

$$\frac{\sec \alpha - \cos \alpha}{\operatorname{cosec} \alpha - \sin \alpha} = \frac{\frac{1}{\cos \alpha} - \cos \alpha}{\frac{1}{\sin \alpha} - \sin \alpha} = \frac{\frac{1 - \cos^2 \alpha}{\cos \alpha}}{\frac{1 - \sin^2 \alpha}{\sin \alpha}} = \frac{\frac{\sin^2 \alpha}{\cos \alpha}}{\frac{\cos^2 \alpha}{\sin \alpha}} = \frac{\sin^3 \alpha}{\cos^3 \alpha} = \tan^3 \alpha$$

e) $(\sin \alpha + \cos \alpha)^2 = 1 + 2 \tan \alpha \cos^2 \alpha$

$$(\sin \alpha + \cos \alpha)^2 = \sin^2 \alpha + 2 \sin \alpha \cos \alpha + \cos^2 \alpha = 1 + 2 \sin \alpha \cos \alpha$$

↓
producto notable

$$1 + 2 \tan \alpha \cos^2 \alpha = 1 + 2 \frac{\sin \alpha}{\cos \alpha} \cdot \cos^2 \alpha = 1 + 2 \sin \alpha \cos \alpha$$

Desarrollando a ambos lados nos

$$f) \operatorname{sen}^2 \alpha = \frac{1}{1 + \cot^2 \alpha}$$

$$\frac{1}{1 + \cot^2 \alpha} = \frac{1}{1 + \frac{\cos^2 \alpha}{\operatorname{sen}^2 \alpha}} = \frac{1}{\frac{\operatorname{sen}^2 \alpha + \cos^2 \alpha}{\operatorname{sen}^2 \alpha}} = \frac{\operatorname{sen}^2 \alpha}{1} = \operatorname{sen}^2 \alpha$$

$$g) 1 + \frac{1}{\operatorname{tg}^2 \alpha} = \frac{1}{\operatorname{sen}^2 \alpha}$$

$$1 + \frac{1}{\frac{\operatorname{sen}^2 \alpha}{\cos^2 \alpha}} = 1 + \frac{\cos^2 \alpha}{\operatorname{sen}^2 \alpha} = \frac{\operatorname{sen}^2 \alpha + \cos^2 \alpha}{\operatorname{sen}^2 \alpha} = \frac{1}{\operatorname{sen}^2 \alpha}$$

$$h) \frac{(1 - \cos \alpha)(1 + \cos \alpha)}{\operatorname{sen} \alpha} = \operatorname{sen} \alpha$$

$$\frac{(1 - \cos \alpha)(1 + \cos \alpha)}{\operatorname{sen} \alpha} = \frac{1 - \cos^2 \alpha}{\operatorname{sen} \alpha} = \frac{\operatorname{sen}^2 \alpha}{\operatorname{sen} \alpha} = \operatorname{sen} \alpha$$

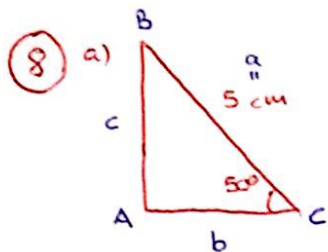
$$i) \frac{\cos^2 \alpha}{1 - \operatorname{sen} \alpha} = 1 + \operatorname{sen} \alpha$$

$$\frac{\cos^2 \alpha}{1 - \operatorname{sen} \alpha} = \frac{1 - \operatorname{sen}^2 \alpha}{1 - \operatorname{sen} \alpha} = \frac{(1 + \operatorname{sen} \alpha)(1 - \operatorname{sen} \alpha)}{1 - \operatorname{sen} \alpha} = 1 + \operatorname{sen} \alpha$$

$$j) \frac{\operatorname{sen} \alpha - \operatorname{sen}^3 \alpha}{\cos \alpha - \cos^3 \alpha} \cdot \operatorname{tg} \alpha = 1$$

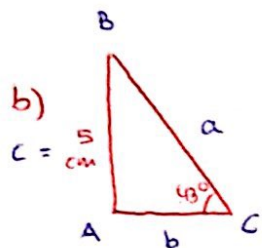
$$\frac{\operatorname{sen} \alpha - \operatorname{sen}^3 \alpha}{\cos \alpha - \cos^3 \alpha} \cdot \operatorname{tg} \alpha = \frac{\operatorname{sen} \alpha (1 - \operatorname{sen}^2 \alpha)}{\cos \alpha (1 - \cos^2 \alpha)} \cdot \frac{\operatorname{sen} \alpha}{\cos \alpha} = \frac{\operatorname{sen}^2 \alpha (1 - \operatorname{sen}^2 \alpha)}{\cos^2 \alpha (1 - \cos^2 \alpha)} =$$

$$= \frac{\operatorname{sen}^2 \alpha \cdot \cos^2 \alpha}{\cos^2 \alpha \cdot \operatorname{sen}^2 \alpha} = 1$$



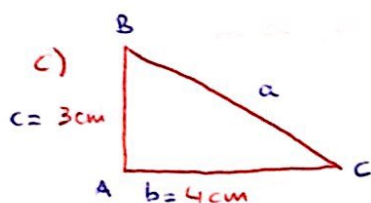
- $\sin 50^\circ = \frac{c}{5} \rightarrow c = \sin 50^\circ \cdot 5 = 3,83 \text{ cm}$
- $b = \sqrt{5^2 - 3,83^2} = 3,21 \text{ cm}$
- $\hat{B} = 180^\circ - 90^\circ - 50^\circ = 40^\circ$

$a = 5 \text{ cm}$
 $b = 3,21 \text{ cm}$
 $c = 3,83 \text{ cm}$
 $\hat{A} = 90^\circ$
 $\hat{B} = 40^\circ$
 $\hat{C} = 50^\circ$



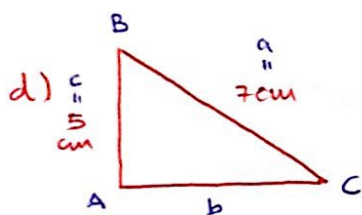
- $\sin 43^\circ = \frac{5}{a} \rightarrow a = \frac{5}{\sin 43^\circ} = 7,33 \text{ cm}$
- $b = \sqrt{7,33^2 - 5^2} = 5,36 \text{ cm}$
- $\hat{B} = 180^\circ - 90^\circ - 43^\circ = 47^\circ$

$a = 7,33 \text{ cm}$
 $b = 5,36 \text{ cm}$
 $c = 5 \text{ cm}$
 $\hat{A} = 90^\circ$
 $\hat{B} = 47^\circ$
 $\hat{C} = 43^\circ$



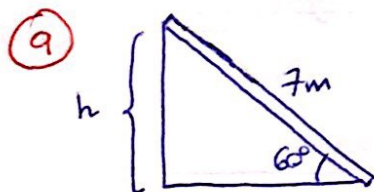
- $a = \sqrt{3^2 + 4^2} = 5 \text{ cm}$
- $\tan \hat{C} = \frac{3}{4} \Rightarrow \hat{C} = \arctan\left(\frac{3}{4}\right) = 36,87^\circ$
- $\hat{B} = 180^\circ - 90^\circ - 36,87^\circ = 53,13^\circ$

$a = 5 \text{ cm}$
 $b = 4 \text{ cm}$
 $c = 3 \text{ cm}$
 $\hat{A} = 90^\circ$
 $\hat{B} = 53,13^\circ$
 $\hat{C} = 36,87^\circ$

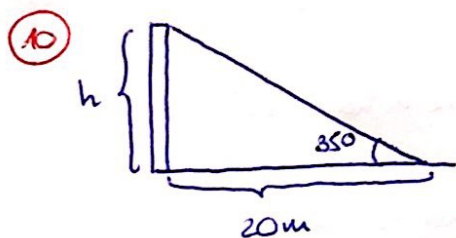


- $b = \sqrt{7^2 - 5^2} = 4,9 \text{ cm}$
- $\sin \hat{C} = \frac{5}{7} \rightarrow \hat{C} = \arcsin\left(\frac{5}{7}\right) = 45,58^\circ$
- $\hat{B} = 180^\circ - 90^\circ - 45,58^\circ = 44,42^\circ$

$a = 7 \text{ cm}$
 $b = 4,9 \text{ cm}$
 $c = 5 \text{ cm}$
 $\hat{A} = 90^\circ$
 $\hat{B} = 44,42^\circ$
 $\hat{C} = 45,58^\circ$

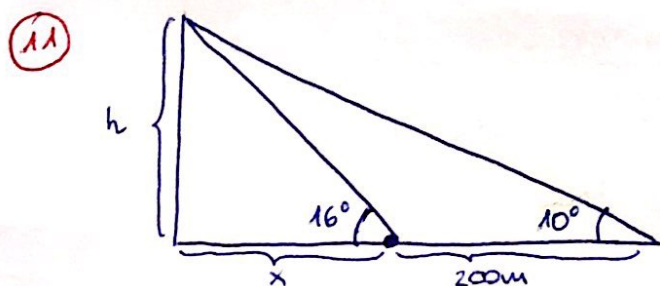


$$\sin 60^\circ = \frac{7}{h} \rightarrow \frac{\sqrt{3}}{2} = \frac{7}{h} \rightarrow h = 8,08 \text{ m}$$



$$\tan 35^\circ = \frac{h}{20} \rightarrow h = \tan 35^\circ \cdot 20$$

$$h = 14 \text{ m}$$



$$\tan 16^\circ = \frac{h}{x} \rightarrow h = x \cdot \tan 16^\circ$$

$$\tan 10^\circ = \frac{h}{x+200} \rightarrow h = (x+200) \cdot \tan 10^\circ$$

$$x + \operatorname{tg} 16^\circ = (x + 200) \operatorname{tg} 10^\circ$$

$$x + \operatorname{tg} 16^\circ = x \operatorname{tg} 10^\circ + 200 \cdot \operatorname{tg} 10^\circ$$

$$x + \operatorname{tg} 16^\circ - x \operatorname{tg} 10^\circ = 200 \cdot \operatorname{tg} 10^\circ$$

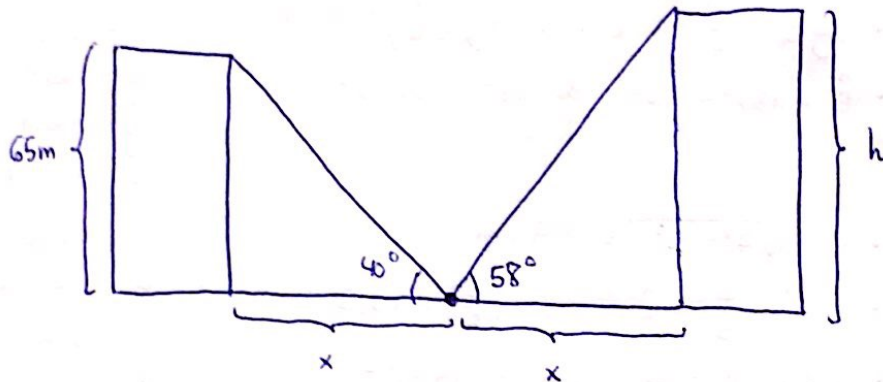
$$x = \frac{200 \cdot \operatorname{tg} 10^\circ}{\operatorname{tg} 16^\circ - \operatorname{tg} 10^\circ} = 319,38 \text{ m}$$

$$h = 319,38 \cdot \operatorname{tg} 16^\circ = 91,58 \text{ m}$$

Solución: Altura del fuste: 91,58 m

Distancia al 2º lugar de observación: 319,38 m

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$$\operatorname{tg} 40^\circ = \frac{65}{x} \rightarrow x = \frac{65}{\operatorname{tg} 40^\circ}$$

$$\operatorname{tg} 58^\circ = \frac{h}{x} \rightarrow x = \frac{h}{\operatorname{tg} 58^\circ}$$

$$\left. \begin{array}{l} \operatorname{tg} 40^\circ = \frac{65}{x} \\ \operatorname{tg} 58^\circ = \frac{h}{x} \end{array} \right\} \frac{65}{\operatorname{tg} 40^\circ} = \frac{h}{\operatorname{tg} 58^\circ}$$

$$\Rightarrow h = \frac{65 \cdot \operatorname{tg} 58^\circ}{\operatorname{tg} 40^\circ} = 123,97 \text{ m.}$$

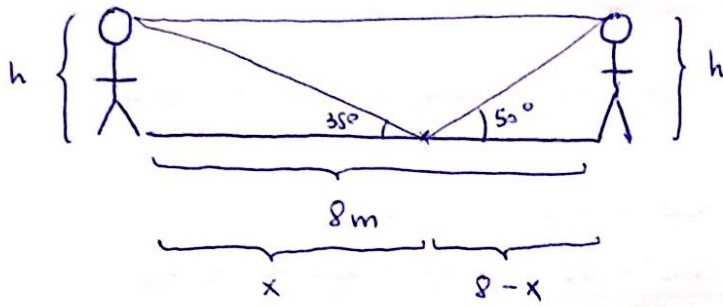
$$\Rightarrow x = \frac{65}{\operatorname{tg} 40^\circ} = 77,46 \text{ m}$$

Solución: La altura del edificio de la derecha es 123,97 m.

La distancia entre ambos edificios es

$$77,46 \cdot 2 = 154,92 \text{ m.}$$

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$$\operatorname{tg} 35^\circ = \frac{h}{x} \longrightarrow h = x \cdot \operatorname{tg} 35^\circ$$

$$\operatorname{tg} 50^\circ = \frac{h}{8-x} \longrightarrow h = (8-x) \cdot \operatorname{tg} 50^\circ$$

$$\Rightarrow x \cdot \operatorname{tg} 35^\circ = (8-x) \operatorname{tg} 50^\circ$$

$$x \cdot \operatorname{tg} 35^\circ = 8 \cdot \operatorname{tg} 50^\circ - x \operatorname{tg} 50^\circ$$

$$x \operatorname{tg} 35^\circ + x \operatorname{tg} 50^\circ = 8 \cdot \operatorname{tg} 50^\circ$$

$$x = \frac{8 \cdot \operatorname{tg} 50^\circ}{\operatorname{tg} 35^\circ + \operatorname{tg} 50^\circ} = 5,04 \text{ m.}$$

Solución: El niño de la derecha se queda con la moneda al estar a 2,96 m.

$$h = 5,04 \cdot \operatorname{tg} 36^\circ = 3,66 \text{ m}$$

Solución: Los niños miden 3,66 m (cómo crecen los niños hoy en día).

14) a) $2 \cos x - 2 = 0 \Rightarrow 2 \cos x = 2 \Rightarrow \cos x = 1$

$$x = 0^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

b) $\sin 2x = \cos 60^\circ \Rightarrow \sin 2x = \frac{1}{2} \Rightarrow 2x = 30^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$

$$x = 15^\circ + 180^\circ \cdot k, k \in \mathbb{Z}$$

La otra solución es: $2x = 180^\circ - 30^\circ = 150^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$

$$x = 75^\circ + 180^\circ \cdot k, k \in \mathbb{Z}$$

c) $3 \cos^2 x = \sin^2 x \Rightarrow 3 \cos^2 x = 1 - \cos^2 x \Rightarrow 3 \cos^2 x + \cos^2 x = 1$

$$\Rightarrow 4 \cos^2 x = 1 \Rightarrow \cos^2 x = \frac{1}{4} \Rightarrow \cos x = \pm \frac{1}{2}$$

• Si $\cos x = \frac{1}{2}$ \rightarrow $x = 60^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$
 $x = 360^\circ - 60^\circ = 300^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$

• Si $\cos x = -\frac{1}{2}$ \rightarrow $x = 120^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$
 $x = 360^\circ - 120^\circ = 240^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$

d) $\sin x + \operatorname{cosec} x = \frac{5}{2}$

$\sin x + \frac{1}{\sin x} = \frac{5}{2}$ $\xrightarrow[\text{ecuación por } \sin x]{\text{multiplico toda la}}$ $\sin^2 x + 1 = \frac{5 \sin x}{2} \Rightarrow$

$\Rightarrow 2 \sin^2 x + 2 = 5 \sin x \Rightarrow 2 \sin^2 x - 5 \sin x + 2 = 0$

$\sin x = \frac{5 \pm \sqrt{25 - 4 \cdot 2 \cdot 2}}{4} = \frac{5 \pm 3}{4} < \frac{2}{\frac{2}{4} = \frac{1}{2}} \rightarrow$ no válida

Si $\sin x = \frac{1}{2} \Rightarrow x = 30^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$
 $x = 180^\circ - 30^\circ = 150^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$

e) $2 \cos x = 3 \operatorname{tg} x \rightarrow 2 \cos x = 3 \frac{\sin x}{\cos x} \rightarrow 2 \cos^2 x = 3 \sin x \rightarrow$

$\rightarrow 2(1 - \sin^2 x) = 3 \sin x \rightarrow 2 - 2 \sin^2 x = 3 \sin x \rightarrow 2 \sin^2 x + 3 \sin x - 2 = 0$

$\sin x = \frac{-3 \pm \sqrt{9 - 4 \cdot 2 \cdot (-2)}}{4} = \frac{-3 \pm 5}{4} < \frac{-2}{\frac{2}{4} = \frac{1}{2}} \rightarrow$ no válida

$\sin x = \frac{1}{2} \rightarrow x = 30^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$
 $x = 150^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$

f) $\sin^2 x - \cos^2 x = \frac{1}{2} \Rightarrow 1 - \cos^2 x - \cos^2 x = \frac{1}{2} \rightarrow -2 \cos^2 x = \frac{1}{2} - 1$

$\rightarrow -2 \cos^2 x = -\frac{1}{2} \rightarrow \cos^2 x = \frac{1}{4} \rightarrow \cos x = \pm \frac{1}{2} = \frac{\sqrt{2}}{2}$

• Si $\cos x = \frac{1}{2}$ \rightarrow $x = 60^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$
 $x = 360^\circ - 60^\circ = 300^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$

• Si $\cos x = -\frac{1}{2}$

$$x = 120^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

$$x = 360^\circ - 120^\circ = 240^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

g) $7 \sin x + 4 \cos^2 x - 2 = 0 \rightarrow 7 \sin x + 4(1 - \sin^2 x) - 2 = 0$

$\rightarrow 7 \sin x + 4 - 4 \sin^2 x - 2 = 0 \rightarrow -4 \sin^2 x + 7 \sin x + 2 = 0$

$$\sin x = \frac{-7 \pm \sqrt{49 - 4 \cdot (-4) \cdot 2}}{-8} = \frac{-7 \pm 9}{-8} \begin{cases} \frac{2}{-8} = -\frac{1}{4} \\ 2 \rightarrow \text{no válida.} \end{cases}$$

Si $\sin x = -\frac{1}{4}$

$$x = -14,48^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

↓ lo paso a un ángulo positivo (sumo 360°)

$$x = 345,52^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

$$x = 180^\circ - 345,52^\circ = -165,52^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

↓ ángulo positivo

$$x = 194,48^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

h) $5 \sec x - 4 \cos x = 8$

$$5 \frac{1}{\cos x} - 4 \cos x = 8 \xrightarrow{\text{multiplico por } \cos x} 5 - 4 \cos^2 x = 8 \cos x \rightarrow$$

$\rightarrow 4 \cos^2 x + 8 \cos x - 5 = 0$

$$\cos x = \frac{-8 \pm \sqrt{64 - 4 \cdot 4 \cdot (-5)}}{8} = \frac{-8 \pm 12}{8} \begin{cases} \frac{4}{8} = \frac{1}{2} \\ -\frac{20}{8} \rightarrow \text{no válida} \end{cases}$$

Si $\cos x = \frac{1}{2}$

$$x = 60^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

$$x = 360^\circ - 60^\circ = 300^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

i) $\sin(2x + 1) = \frac{1}{2}$

$$2x + 1 = 30^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

$$2x = 29^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

$$x = 14,5^\circ + 180^\circ \cdot k, k \in \mathbb{Z}$$

$$2x + 1 = 180^\circ - 30^\circ = 150^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

$$2x = 149^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

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$$2x + 1 = 30^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

$$2x = 29^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

$$\boxed{x = 14,5^\circ + 180^\circ \cdot k, k \in \mathbb{Z}}$$

$$2x + 1 = 180^\circ - 30^\circ = 150^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

$$2x = 149^\circ + 360^\circ k, k \in \mathbb{Z} \rightarrow \boxed{x = 74,5^\circ + 180^\circ k, k \in \mathbb{Z}}$$

⑥