FICHA REPASO NÚMEROS COMPLEJOS

1\. a) $(z_1 - z_2) \cdot z_3$

$(-3+4i-(5-2i)) \cdot 7i = (-3+4i-5+2i) \cdot 7i =$

$= (-8+6i) \cdot 7i = -56i + 42i^2 = \boxed{-42-56i}$

b) $z_1^2 \cdot z_3$

$(-3+4i)^2 \cdot 7i = (9 - 2 \cdot 3 \cdot 4i + 16i^2) \cdot 7i = (9-24i-16) \cdot 7i =$

$= (-7-24i) \cdot 7i = -49i - 168i^2 = \boxed{168-49i}$

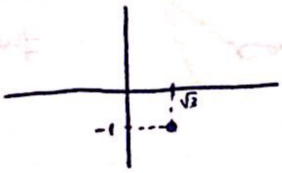
c) $\dfrac{z_2}{z_1} = \dfrac{5-2i}{-3+4i} \cdot \dfrac{-3-4i}{-3-4i} = \dfrac{-15-20i+6i+8i^2}{9+16} = \dfrac{-23-14i}{25} = \boxed{\dfrac{-23}{25} - \dfrac{14}{25}i}$

2\. a) $\sqrt{3} - i = (\sqrt{3}, -1) = 2_{330^\circ} = 2(\cos 330^\circ + i \operatorname{sen} 330^\circ)$

$m = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{3+1} = \sqrt{4} = 2$

$\alpha = \operatorname{arctg}\left(\frac{-1}{\sqrt{3}}\right) = -30^\circ = 330^\circ$

pasamos a un ángulo positivo



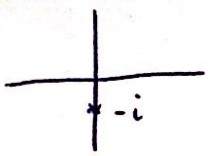
4º cuadrante

b) $4_{60^\circ} = 4(\cos 60^\circ + i \operatorname{sen} 60^\circ) = 4\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) =$

$= 2 + 2\sqrt{3}\,i = (2, 2\sqrt{3})$

$\boxed{\begin{array}{l} \operatorname{sen} 60^\circ = \frac{\sqrt{3}}{2} \\ \cos 60^\circ = 1/2 \end{array}}$

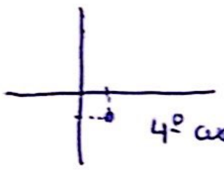
c) $-i = 1_{270^\circ} = \cos 270^\circ + i \operatorname{sen} 270^\circ = (0, -1)$



3\. $\left(\dfrac{1-i}{1+i}\right)^4$

Paso primero a forma polar, porque es más fácil calcular la potencia y el cociente en forma polar.

$1 - i = \sqrt{2}_{315°}$



4º cuadrante

$m = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\alpha = \operatorname{arctg}\left(\frac{-1}{1}\right) = \operatorname{arctg}(-1) = -45° + 360° =$

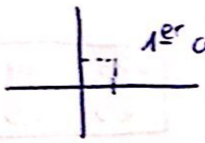
$= 315°$

↓

paso a un

ángulo positivo.

$1 + i = \sqrt{2}_{45°}$



1er cuadrante

$m = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\alpha = \operatorname{arctg}(1) = 45°$

Ahora calculo:

$$\left(\frac{1-i}{1+i}\right)^4 = \left(\frac{\sqrt{2}_{315°}}{\sqrt{2}_{45°}}\right)^4 = (1_{270°})^4 = 1^4_{4\cdot 270°} = 1_{\underbrace{1080°}_{\text{es mayor que } 360° \ *}} = 1_{0°} = \boxed{1}$$

↓

divido módulos

y resto argumentos

*

$$\begin{array}{r|l} 1080° & 360° \\ \hline 000 & 3 \end{array}$$

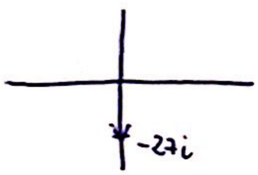
↓

resto $0°$

luego $1080° = 0°$

4. $\sqrt[3]{-27i}$ Paso a forma polar: $-27i = 27_{270°}$

Se ve gráficamente:



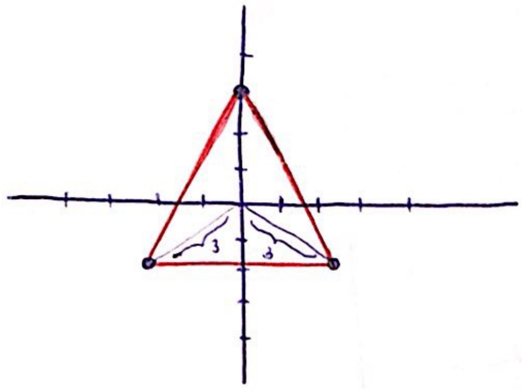
$m = 27$

$\alpha = 270°$

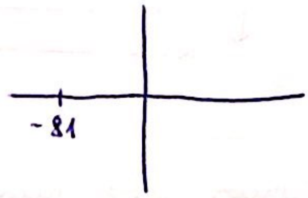
$$\sqrt[3]{-27i} = \sqrt[3]{27_{270°}} = \sqrt[3]{27}_{\frac{270° + 360°\cdot k}{3}} = 3_{90° + 120° k} =$$

$$= \begin{cases} k = 0 \longrightarrow \boxed{3_{90°}} \\ k = 1 \longrightarrow \boxed{3_{210°}} \\ k = 2 \longrightarrow \boxed{3_{330°}} \end{cases}$$

Si las dibujásemos quedaría un triángulo equilátero:



5. $\sqrt[4]{-81}$ Pasamos -81 a forma polar



$$-81 = 81_{180^\circ}$$

$$\sqrt[4]{-81} = \sqrt[4]{81_{180^\circ}} = \sqrt[4]{81}_{\frac{180^\circ + 360^\circ K}{4}} = 3_{45^\circ + 90^\circ K} =$$

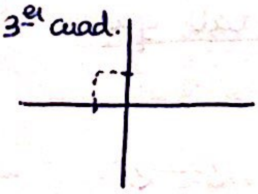
$$= \begin{cases} K=0 \longrightarrow 3_{45^\circ} \\ K=1 \longrightarrow 3_{135^\circ} \\ K=2 \longrightarrow 3_{225^\circ} \\ K=3 \longrightarrow 3_{315^\circ} \end{cases}$$

6. Buscamos un número complejo z que cumple que:

$$\sqrt[8]{z} = -1+i \Rightarrow z = (-1+i)^8$$

Para calcular la potencia octava es mejor pasar a forma polar:

$-1+i \longrightarrow$



$$m = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

$$\alpha = \text{arctg}\left(\frac{-1}{1}\right) = -45^\circ$$

está en el 4º cuadrante

Busco un ángulo en el 3er cuadrante: $-45^\circ + 180^\circ = \underline{\underline{135^\circ}}$

Luego, $(-1+i)^8 = (\sqrt{2}_{135^\circ})^8 = \sqrt{2^8}_{8\cdot 135^\circ} = 16_{1080^\circ} = 16_{0^\circ}$

Luego, $z = 16_{0^\circ}$. El resto de raíces son:

$$\sqrt[8]{16_{0^\circ}} = \sqrt{2}_{\frac{0^\circ + 360^\circ K}{8}} = \sqrt{2}_{45^\circ \cdot K} =$$

$=$
- $K = 0 \longrightarrow \sqrt{2}_{0^\circ}$
- $K = 1 \longrightarrow \sqrt{2}_{45^\circ}$
- $K = 2 \longrightarrow \sqrt{2}_{90^\circ}$
- $K = 3 \longrightarrow \sqrt{2}_{135^\circ}$
- $K = 4 \longrightarrow \sqrt{2}_{180}$
- $K = 5 \longrightarrow \sqrt{2}_{225^\circ}$
- $K = 6 \longrightarrow \sqrt{2}_{270}$
- $K = 7 \longrightarrow \sqrt{2}_{315^\circ}$

7. Buscamos dos números z, z' tales que: $z \cdot z' = 2\sqrt{2}_{75^\circ}$

Vamos a pasar z a su forma polar:

$z = 1 + i = \sqrt{2}_{45^\circ}$ (lo hemos hecho antes)

Será: $\sqrt{2}_{45^\circ} \cdot z' = 2\sqrt{2}_{75^\circ}$

Despejando:

$$z' = \frac{2\sqrt{2}_{75^\circ}}{\sqrt{2}_{45^\circ}} = 2_{30^\circ}$$

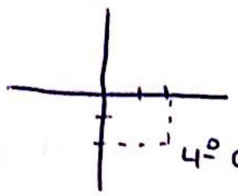
En forma binómica: $2_{30^\circ} = 2(\cos 30^\circ + i \operatorname{sen} 30^\circ) =$

$$= 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \boxed{\sqrt{3} + i}$$

8. Buscamos un número complejo tal que $\sqrt[4]{z} = 2 - 2i$

Será entonces $z = (2-2i)^4$

Pasamos a forma polar: $2 - 2i$



$m = \sqrt{2^2 + (-2)^2} = \sqrt{8}$

$\alpha = \operatorname{arctg}(-1) = -45^\circ + 360^\circ = 315^\circ$

$$z = \left(\sqrt{8}_{315^\circ}\right)^4 = 64_{1260^\circ} = 64_{180^\circ}$$

$$\begin{array}{r|l} 1260^\circ & 360^\circ \\ \hline 180 & 3 \end{array}$$

El resto de raíces será:

$$\sqrt[4]{64_{180^\circ}} = \sqrt{8}_{\frac{180^\circ+360^\circ k}{4}} = \sqrt{8}_{45^\circ+90^\circ k} =$$

$$= \begin{cases} k=0 \longrightarrow \sqrt{8}_{45^\circ} \\ k=1 \longrightarrow \sqrt{8}_{135^\circ} \\ k=2 \longrightarrow \sqrt{8}_{225^\circ} \\ k=3 \longrightarrow \sqrt{8}_{315^\circ} \end{cases}$$

9. a) $z^2 - 4z + 5 = 0$

$$z = \frac{4 \pm \sqrt{16-4\cdot1\cdot5}}{2} = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm \sqrt{4\cdot(-1)}}{2} = \frac{4 \pm \sqrt{4}\cdot\sqrt{-1}}{2} =$$

$$= \frac{4 \pm 2i}{2} = 2 \pm i \begin{cases} 2+i \\ 2-i \end{cases}$$

b) $z^3 + 8 = 0 \longrightarrow z^3 = -8 \longrightarrow z = \sqrt[3]{-8} \longrightarrow$

$$\longrightarrow z = \sqrt[3]{8_{180^\circ}} = \sqrt[3]{8}_{\frac{180^\circ+360^\circ k}{3}} = \sqrt[3]{8}_{60^\circ+120^\circ k} =$$

$$= \begin{cases} k=0 \longrightarrow \sqrt[3]{8}_{60^\circ} \\ k=1 \longrightarrow \sqrt[3]{8}_{180^\circ} \\ k=2 \longrightarrow \sqrt[3]{8}_{300^\circ} \end{cases}$$

c) $z^4 + 13z^2 + 36 = 0 \longrightarrow$ Cambio de variable $z^2 = t$

$t^2 + 13t + 36 = 0$

$$t = \frac{-13 \pm \sqrt{169-4\cdot 1\cdot 36}}{2} = \frac{-13 \pm 5}{2} \begin{cases} -9 \\ -8 \end{cases}$$

Deshacemos cambio:

$$z^2 = -9 \Rightarrow z = \sqrt{-9} = \sqrt{9_{180^\circ}} = 3_{\frac{180^\circ+360^\circ k}{2}} =$$

$$= 3_{90^\circ+180^\circ k} \begin{cases} 3_{90^\circ} = \boxed{3i} \\ 3_{270^\circ} = \boxed{-3i} \end{cases}$$

$$z^2 = -8 \Rightarrow z = \sqrt{-8} = \sqrt{8_{180^\circ}} = \sqrt{8}_{\frac{180^\circ+360^\circ k}{2}} =$$

$$= \sqrt{8}_{90^\circ+180^\circ k} \begin{cases} k=0 \longrightarrow \sqrt{8}_{90^\circ} = \boxed{\sqrt{8}\,i} \\ k=1 \longrightarrow \sqrt{8}_{270^\circ} = \boxed{-\sqrt{8}\,i} \end{cases}$$

d) $z^3+64=0 \longrightarrow z^3 = -64 \longrightarrow z = \sqrt[3]{-64} = \sqrt[3]{64_{180^\circ}} =$

$$= 4_{\frac{180^\circ+360^\circ k}{3}} = 4_{60^\circ+120^\circ k} \begin{cases} k=0 \rightarrow 4_{60^\circ} \\ k=1 \rightarrow 4_{180^\circ} \\ k=2 \rightarrow 4_{300^\circ} \end{cases}$$